1. *Quantum Mechanics* (Spring 2003)

In one-dimension, a particle is subject to a harmonic oscillator potential with a time dependent origin,

$$V(x) = \frac{1}{2}m\omega^2(x - \epsilon(t))^2$$

where

$$\epsilon(t) = \epsilon e^{-t^2/\tau^2}, \quad \epsilon \ll 1$$

Suppose the particle is in the ground state at $t = -\infty$. What states can the particle be in at $t = +\infty$, and what are the probabilities for each? Work to lowest order in ϵ.

2. *Quantum Mechanics* (Spring 2003)

Consider two $s = 1/2$ spins interacting through the Hamiltonian

$$H = J\sigma_1^z\sigma_2^z + h(\sigma_1^x + \sigma_2^x)$$

What is the ground state energy?

3. *Quantum Mechanics* (Spring 2003)

A *coherent state* of a simple harmonic oscillator is an eigenstate of the annihilation operator, a. In terms of the energy eigenvalue basis, give an explicit expression for a coherent state $|\alpha\rangle$ satisfying $a\,|\alpha\rangle = \alpha\,|\alpha\rangle$.

4. *Quantum Mechanics* (Spring 2003)

Consider two electrons which are constrained to live on two sites. There is an interaction energy U when both electrons are on the same site. When they are on different sites, there is no interaction energy. There is an amplitude t for an electron to hop from one site to the other. In other words, the Hamiltonian is of the form:

$$H = -t(|1\uparrow, 1\downarrow\rangle\langle 1\uparrow, 2\downarrow| + \text{h.c.} + |2\uparrow, 2\downarrow\rangle\langle 1\uparrow, 2\downarrow| + \text{h.c.}) + U(|1\uparrow, 1\downarrow\rangle\langle 1\uparrow, 1\downarrow| + |2\uparrow, 2\downarrow\rangle\langle 2\uparrow, 2\downarrow|)$$

where $|1\sigma, 2\sigma'\rangle$ is the state with an electron of spin $\sigma = \uparrow, \downarrow$ at site 1 and an electron of spin $\sigma' = \uparrow, \downarrow$ at site 2 while $|1\sigma, 1\ -\sigma\rangle$ is the state with two electrons (of spins σ and $-\sigma$) at site 1. What are the energies and degeneracies of the ground and first excited states of the system to lowest order in t for $t \ll U$?

5. *Quantum Mechanics* (Spring 2003)

Consider a particle of mass m which moves in the potential

$$V(x) = \begin{cases} \infty & \text{for } x < 0 \\ ax & \text{for } x > 0 \end{cases}$$

Estimate the ground state energy.

6. *Statistical Mechanics and Thermodynamics* (Spring 2003)

Consider a system with N lattice sites and N atoms. These atoms may move around, but each site may be occupied by only 0,1, or 2 atoms at a time. Note that if there are n unoccupied sites, then there must also be n doubly-occupied sites; therefore, macrostates may be denoted by the value of n. The energy of a site is 0 if unoccupied, 0 if singly occupied, and e if doubly occupied; the total energy is thus $U(n) = ne$.

(a) Find the number of microstates vs. n.

(b) Find the temperature using part (a).

(c) What is the grand canonical partition function for a single site? Using this and the constraints of the problem, deduce the chemical potential.

(d) Find the average energy per site using part (c); confirm your answer using part (b).

7. *Statistical Mechanics and Thermodynamics* (Spring 2003)

Consider two atoms, A and B, of mass m_A and m_B. There atoms can form a molecule $C = AB$ with binding energy Δ. Initially, a certain number, N_A and N_B, of A and B atoms is placed in a box of volume V. If the system is brought to thermal equilibrium at temperature T, how many atoms A and B and molecules C will be found in the box? You can assume that A, B, and C are noninteracting (more precisely, the only effect of the interaction is to give rise to the bound state C.) You may also assume a dilute concentration of atoms and molecules.

8. *Electricity and Magnetism* (Spring 2003)

A cylindrical capacitor of length L is composed of an inner cylindrical conductor of radius r and a concentric outer conducting cylindrical shell of radius R.

(a) What is the capacitance of this arrangement (you may ignore fringing fields at the ends)?

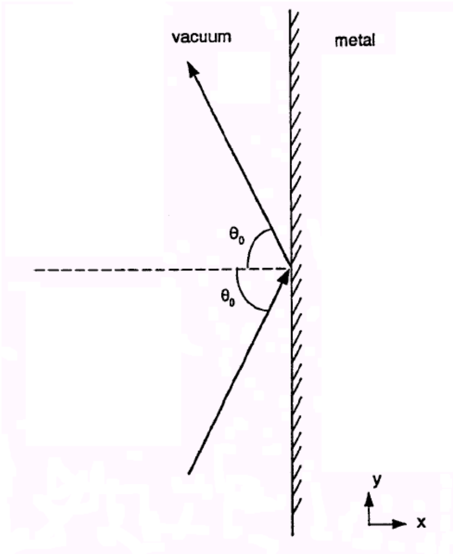
(b) The two conductors are held at a constant potential difference, V, using a battery. A cylindrical shell of dielectric material of length L and which just fits in between the conductors (inner radius $\sim r$ and outer radius $\sim R$) is inserted so that half is inside of the capacitor (i.e. $L/2$ of the length of the capacitor is now filled with dielectric). What is the force on the dielectric in this position (magnitude and direction)?

9. *Electricity and Magnetism* (Spring 2003)

Consider the infinite two-dimensional conducting plane depicted in the figure. The right half is maintained at electrostatic potential V_0 while the left half is maintained at potential $-V_0$. What is the potential above the plane?

10. *Electricity and Magnetism* (Spring 2003)

X-Ray Mirror: X-rays which strike a metal surface at an angle of incidence to the normal greater than a critical angle θ_0 are totally reflected. As shown below, the metal occupies the region $x > 0$. The X-rays propagate in the x–y plane (the plane of the picture) and their polarization is in the z direction, coming out of the page. Assume that the metal contains n free electrons per unit volume and is non-magnetic. Derive an expression for the critical angle θ_0.



11. *Electricity and Magnetism* (Spring 2003)

Secret Circuit: A two-terminal "black box" is given to you. Inside the box a circuit is attached to the terminals which is known to contain a lossless inductor L, a lossless capacitor C, and a resistor R. When a 1.5 Volt battery is connected across the terminals, a current of 1.5 milliamperes flows. When an AC voltage of 1.0 Volt (RMS) at a frequency of 60 Hz is connected, a current of 0.01 amperes (RMS) flows. As the AC frequency is increased while the applied voltage is maintained constant, the current is found to go through a maximum exceeding 100 amperes at $\nu = 1000$ Hz. What is the circuit inside the box? What are the values of R, L, and C?

12. *Electricity and Magnetism* (Spring 2003)

(a) Show that the field inside a sphere of uniformly magnetized material ($\mathbf{M} = \mathrm{M}\ \hat{\mathbf{z}}$) is:

$$\mathbf{B} = \frac{2}{3}\mu_0 M\hat{\mathbf{z}}$$

(b) A sphere of material with linear magnetic susceptibility χ_m is placed in a region of uniform magnetic field $B_o\hat{\mathbf{z}}$. Using the above result, find the magnetic field inside the sphere.

13. *Statistical Mechanics and Thermodynamics* (Spring 2003)

Consider a d-dimensional material in which the important excitation are non-conserved bosons, and assume that the dispersion relation for these bosons is $\omega = ak^3$, where k is the wave vector's amplitude and a is a constant. The low temperature specific heat goes as T^q. What is the value of the power, q? Note: The dimensionality, d, of the material is not necessarily equal to three.

14. *Statistical Mechanics and Thermodynamics* (Spring 2003)

A system can exchange energy and volume with a large reservoir.

(a) Show that the entropy of this combined system (system & reservoir) is maximized when the temperature of the system is equal to the temperature of the reservoir and the pressure of the system is equal to the pressure of the reservoir.

(b) Assume that the reservoir is much larger than the system. Expand to second order in the energy and volume of the system. Find the inequalities which must be satisfied in order that the entropy of the combined system is a maximum at the extremum point.