11. *Electricity and Magnetism* (Fall 2003)

The dispersion relation for a photon in an ionized plasma (in *CGS* units) is,

$$k^2c^2 = \omega^2 - 4\pi ne^2/m_e$$

where k is the photon wavenumber, $c = 3.0 \times 10^{10}$ cm/s, and ω is the radiation frequency in radians/s. Here, n is the electron number density, $e = 4.8 \times 10^{-10}$ esu is the electron charge, and $m_e = 9.11 \times 10^{-28}$ g is the electron mass.

(a) *Explain* why electromagnetic waves with frequencies below about 10 MHz can't be received from space on Earth.

(b) Pulsars are objects observed in our galaxy which regularly emit a short burst of electromagnetic waves containing a wide range of frequencies all at once. If a pulsar is located 1.0×10^{22} cm away and the density of electrons in the space between us and the pulsar is a uniform 0.01 cm^{-3}, what is the difference of the arrival times at Earth of the radiation emitted near 6 kHz compared to 10 kHz? (You may assume the measurement happens far enough above the Earth so that the effect in part (a) can be ignored. You may leave your answer as an expression without substituting the numbers.)

a. For low frequnencies the value of the dispersion relation will become negative, which means that k becomes imaginary, corresponding to evanescent waves which have an exponentially decaying amplitude.

b. Let $L = 1.0 \times 10^{22}$ cm, $f_1 = 6$ KHz, $f_2 = 10$ KHz, $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, $n = 0.01$ cm^{-3}

$t_1 = \frac{L}{v_1} = \frac{L}{c} n_1$ and $t_2 = \frac{L}{v_2} = \frac{L}{c} n_2$

where $n_i = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} \cong \sqrt{\frac{\varepsilon(\omega)}{\varepsilon_0}}$ since $\mu \cong \mu_0$

But $v = \frac{\omega}{k} \Rightarrow n_i = \sqrt{\frac{c^2}{v^2}} = \sqrt{\frac{k^2c^2}{\omega^2}} = \sqrt{1 - \frac{4\pi ne^2}{\omega^2 m_e}} =$

Therefore $\Delta t = |t_2 - t_1| = \left| \sqrt{1 - \frac{4\pi ne^2}{\omega_2^2 m_e}} - \sqrt{1 - \frac{4\pi ne^2}{\omega_1^2 m_e}} \right|$