11. *Electricity and Magnetism* (Fall 2004)

Using general principles, find the radiated power in vacuum of a non-relativistic point charge q whose position is $\mathbf{r}(t)$. You do not need to find dimensionless proportionality constants (i.e., only find the dependence on q, $\mathbf{r}(t)$, and universal constants).

We will use the general principle that the radiation field is an acceleration field that goes like $\frac{1}{r}$

$\Rightarrow \vec{E}_a \propto \frac{b}{4\pi\epsilon_0}\frac{e}{r}a$ where $a = |\ddot{\vec{r}}(t)|$ is the acceleration and b is a constant of unknown dimension.

$$[\vec{E}] = \left[\frac{1}{4\pi\epsilon_0}\frac{e}{r^2}\right] = [\vec{E}_a] \Rightarrow \left[\frac{1}{4\pi\epsilon_0}\frac{e}{r^2}\right] = \left[\frac{b}{4\pi\epsilon_0}\frac{e}{r}a\right]$$

$$\Rightarrow [b] = \left[\frac{1}{ra}\right] = \frac{s^2}{m^2} \Rightarrow b \propto \frac{1}{c^2}$$

$$\Rightarrow \vec{E}_a \propto \frac{ea}{\epsilon_0 c^2 r}$$

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0 c}|\vec{E}_a|^2\hat{k}$ since $E = cB$ and $\hat{E}\times\hat{B} = \hat{k}$ and $\vec{E}\perp\vec{B}$

$\Rightarrow \vec{S} \propto \frac{1}{\mu_0 c}\frac{e^2a^2}{\epsilon_0^2 c^4 r^2}\hat{k} = \frac{\mu_0^2\cancel{\epsilon_0^2}}{\mu_0 c}\frac{e^2a^2}{\cancel{\epsilon_0^2}r^2}\hat{k} = \frac{\mu_0}{c}\frac{e^2a^2}{r^2}\hat{k}$ since $c^2 = \frac{1}{\mu_0\epsilon_0}$

$\frac{dP}{d\Omega} = \frac{1}{2}\mathrm{Re}[r^2\vec{S}\cdot\hat{n}] \propto \frac{1}{2}\mathrm{Re}\left[\frac{\mu_0}{c}e^2a^2\cos(\theta)\right]$ by taking $\hat{z} = \hat{k}$

$$\Rightarrow \frac{dP}{d\Omega} \propto \frac{\mu_0}{c}e^2a^2\cos(\theta)$$

$$P = \int\frac{dP}{d\Omega}d\Omega \propto \frac{\mu_0}{c}e^2a^2\int_0^{\pi}\cos(\theta)\sin(\theta)d\theta\int_0^{2\pi}d\phi \propto \frac{\mu_0}{c}e^2a^2$$