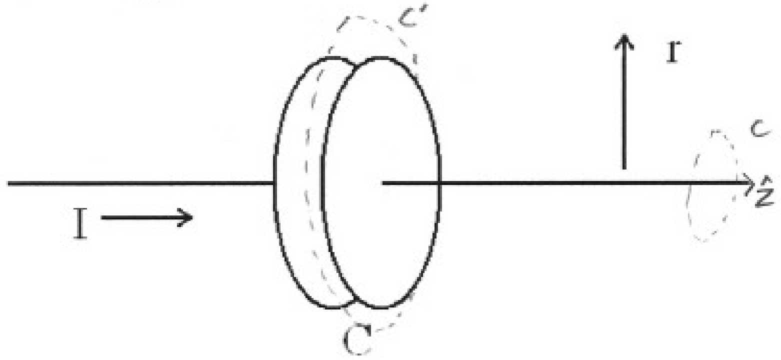
9. *Electricity and Magnetism* (Fall 2004)

A wire carrying current I is connected to a circular capacitor of capacitance C, as depicted in the figure. What is the magnetic field outside the wire, far from the capacitor (as a function of the distance r from the wire)? Using Maxwell's equations, explain why there is a magnetic field outside the capacitor. What is this magnetic field?



Far from the capacitor it looks like a regular current carrying wire, so using Ampere's Law, $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t}$

$$\Rightarrow \int_S(\vec{\nabla}\times\vec{B})\cdot d\vec{a} = \mu_0\int_S \vec{J}\cdot d\vec{a} + \mu_0\epsilon_0\int_S \frac{\partial \vec{E}}{\partial t}\cdot d\vec{a}$$

$$\Rightarrow \int_c \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\epsilon_0\int_S \cancel{\frac{\partial \vec{E}}{\partial t}}\cdot d\vec{a} \qquad \text{since } \vec{E}=0$$

$$\Rightarrow 2\pi r B = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r} \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

Outside the capacitor, we can get the field on C' by integrating over a surface that balloons out around the plates to intersect the wire and we get the same answer. If we choose to use the minimal surface spanning C', then

$$\int_c \vec{B}\cdot d\vec{l} = \cancel{\mu_0 I} + \mu_0\epsilon_0\int_S \frac{\partial \vec{E}}{\partial t}\cdot d\vec{a} \qquad \text{since } I=0$$

$$2\pi r B = \mu_0\cancel{\epsilon_0}\int_S \frac{\partial}{\partial t}\left(\frac{\sigma}{\cancel{\epsilon_0}}\hat{z}\right)\cdot d\vec{a}$$

$$2\pi r B = \mu_0\frac{\partial}{\partial t}\left(\frac{Q}{A}\right)\int_S da = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r} \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

The field has the same expression outside the capacitor because the changing electric field creates a displacement current.