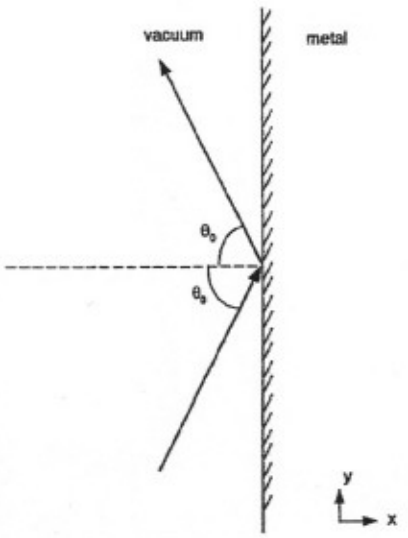
10. *Electricity and Magnetism* (Spring 2003)

X-Ray Mirror: X-rays which strike a metal surface at an angle of incidence to the normal greater than a critical angle θ_0 are totally reflected. As shown below, the metal occupies the region $x > 0$. The X-rays propagate in the x–y plane (the plane of the picture) and their polarization is in the z direction, coming out of the page. Assume that the metal contains n free electrons per unit volume and is non-magnetic. Derive an expression for the critical angle θ_0.



The critical angle comes from Snell's Law when $\theta_2 = \frac{\pi}{2}$

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \rightarrow n_2$$

$$\Rightarrow \sin(\theta_1) = \frac{n_2}{n_1} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\Rightarrow \theta_c = \sin^{-1}(n_2) \quad \text{since } n_1 = 1 \text{ in vacuum}$$

The index of refraction of the metal is calculated from the plasma frequency in the high frequency approximation since X-rays are high frequency.

$$v = \frac{c}{n} \Rightarrow n = \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} = \sqrt{\frac{\epsilon}{\epsilon_0}} \quad \text{since the metal is non-magnetic}$$

$$\Rightarrow n \cong \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \quad \text{where } \omega_p^2 = \frac{ne^2}{\epsilon_0 m}$$

$$\Rightarrow \theta_c \cong \sin^{-1}(n_2) \cong \sin^{-1}\left(\sqrt{1 - \frac{ne^2}{\epsilon_0 m \omega^2}}\right)$$