5. *Quantum Mechanics* (Spring 2003)

Consider a particle of mass m which moves in the potential

$$V(x) = \begin{cases} \infty & \text{for } x < 0 \\ ax & \text{for } x > 0 \end{cases}$$

Estimate the ground state energy.

see Griffiths Example 8.3

Perhaps you could use the variational method for this problem, but it lends itself more to the WKB approximation.

Recall the formula for the WKB approximation in a well with one infinite wall: $\int_0^{x_2} \sqrt{2m(E-V(x))}\,dx = (n-\frac{1}{4})\pi\hbar \quad (n \in \mathbb{Z}^+)$

where x_2 is the classical turning point: $E = V(x_2) \Rightarrow x_2 = \frac{E}{a}$

$$\Rightarrow \int_0^{E/a} \sqrt{2m(E_n - ax)}\,dx = (n-\tfrac{1}{4})\pi\hbar$$

Let $u = 2m(E_n - ax) \Rightarrow du = -2ma\,dx$

$$\int_{x=0}^{x=E/a} u^{1/2} \frac{du}{-2ma} = \frac{1}{-2ma}\left(\frac{2}{3}u^{3/2}\right)\Big|_{x=0}^{x=E/a}$$

$$= \frac{1}{-3ma}\left(2m(E_n - ax)\right)^{3/2}\Big|_{x=0}^{x=E/a} = \frac{1}{3ma}(2mE)^{3/2}$$

$$\Rightarrow \frac{1}{3ma}(2mE_n)^{3/2} = (n-\tfrac{1}{4})\pi\hbar$$

$$\Rightarrow (2mE_n)^{3/2} = 3ma(n-\tfrac{1}{4})\pi\hbar$$

$$\Rightarrow E_n = \frac{1}{2m}\left[3ma(n-\tfrac{1}{4})\pi\hbar\right]^{2/3}$$

So the ground state energy is approximately

$$E_1 = \frac{1}{2m}\left(\frac{9}{4}ma\pi\hbar\right)^{2/3}$$