1. *Quantum Mechanics* (Spring 2004)

The table below shows some Clebsch-Gordan coefficients. If two particles have spin 1/2 and 3/2 respectively, write down all composite states $|sm\rangle$ in terms of the uncoupled states using Dirac notation. You may use the following table if you wish. (A square root is understood for all entries in the table below, with the ± sign outside the radical.)

Notation:

		J	J ...
		M	M ...
m_1	m_2	Coefficients	
m_1	m_2		
⋮	⋮		

$3/2 \times 1/2$

m_1	m_2	$J=2$, $M=+2$	$J=2$, $M=+1$	$J=1$, $M=+1$	$J=2$, $M=0$	$J=1$, $M=0$	$J=2$, $M=-1$	$J=1$, $M=-1$	$J=2$, $M=-2$
+3/2	+1/2	1							
+3/2	−1/2		1/4	3/4					
+1/2	+1/2		3/4	−1/4					
+1/2	−1/2				1/2	1/2			
−1/2	+1/2				1/2	−1/2			
−1/2	−1/2						3/4	1/4	
−3/2	+1/2						1/4	−3/4	
−3/2	−1/2								1

$S_1 = \frac{1}{2} \quad S_2 = \frac{3}{2} \quad |\frac{1}{2} - \frac{3}{2}| \leq S \leq \frac{1}{2} + \frac{3}{2} \Rightarrow 1 \leq S \leq 2$ and $|m| \leq S$

Uncoupled States $|s_1 s_2\, m_1 m_2\rangle$

$|\frac{1}{2}\ \frac{3}{2}\rangle \quad |-\frac{1}{2}\ \frac{3}{2}\rangle$

$|\frac{1}{2}\ \frac{1}{2}\rangle \quad |-\frac{1}{2}\ \frac{1}{2}\rangle$

$|\frac{1}{2}\ -\frac{1}{2}\rangle \quad |-\frac{1}{2}\ -\frac{1}{2}\rangle$

$|\frac{1}{2}\ -\frac{3}{2}\rangle \quad |-\frac{1}{2}\ -\frac{3}{2}\rangle$

Composite States $|s_1 s_2\, m s\rangle$

$|2\ 2\rangle$

$|1\,1\rangle \quad |2\ 1\rangle$

$|1\,0\rangle \quad |2\ 0\rangle$

$|1\,-1\rangle \quad |2\ -1\rangle$

$|2\ -2\rangle$

The table is really a chain of tables stacked corner-to-corner.

$$|2\ 2\rangle = |\tfrac{3}{2}\ \tfrac{1}{2}\rangle$$

$$|2\ 1\rangle = \sqrt{\tfrac{1}{4}}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{3}{4}}\,|\tfrac{1}{2}\ \tfrac{1}{2}\rangle$$

$$|2\ 0\rangle = \sqrt{\tfrac{1}{2}}\,|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{2}}\,|-\tfrac{1}{2}\ \tfrac{1}{2}\rangle$$

$$|2\ -1\rangle = \sqrt{\tfrac{3}{4}}\,|-\tfrac{1}{2}\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{4}}\,|-\tfrac{3}{2}\ \tfrac{1}{2}\rangle$$

$$|2\ -2\rangle = |-\tfrac{3}{2}\ -\tfrac{1}{2}\rangle$$

$$|1\ 1\rangle = \sqrt{\tfrac{3}{4}}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{4}}\,|\tfrac{1}{2}\ \tfrac{1}{2}\rangle$$

$$|1\ 0\rangle = \sqrt{\tfrac{1}{2}}\,|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{2}}\,|-\tfrac{1}{2}\ \tfrac{1}{2}\rangle$$

$$|1\ -1\rangle = \sqrt{\tfrac{1}{4}}\,|-\tfrac{1}{2}\ -\tfrac{1}{2}\rangle - \sqrt{\tfrac{3}{4}}\,|-\tfrac{3}{2}\ \tfrac{1}{2}\rangle$$