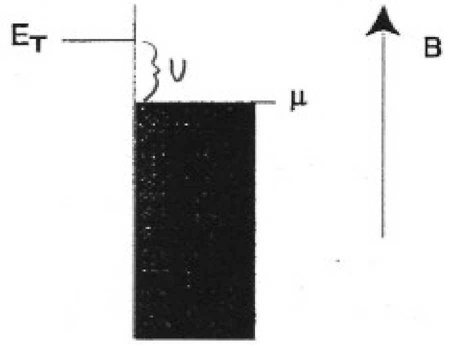
7. *Statistical Mechanics and Thermodynamics* (Spring 2004)

A quantum state at energy E_T is embedded in a system with a degenerate Fermi gas as, for instance, occurs with an impurity state with energy E_T in a degenerate semiconductor with a sea of conducting electrons at chemical potential μ. You may assume that $E_T > \mu$. The impurity, which has a spin of 1/2, can take an additional electron from the large bath of electrons (costs Coulomb energy U), to form a spin-singlet state. For a given temperature T and magnetic field B, calculate the ratio of the probability for the trap being empty to that for the trap being filled by an additional electron.



When the trap is empty it has energy associated with the spin of $\frac{\hbar}{2}$ interacting with the magnetic field. When the trap is filled, the total spin is zero, so there is no interaction with the magnetic field, but it has energy $U = E_T - \mu$. $\vec{\mu} = g\mu_B \vec{s}$ where $\hbar\vec{s}$ is the spin

$$E_B = -\vec{\mu}\cdot\vec{B} = -\tfrac{1}{2} g\mu_B B$$

Therefore $E_{empty} = -\frac{1}{2} g\mu_B B$ and $E_{filled} = U$.

$$P_{empty} = \frac{e^{-\beta E_{empty}}}{e^{-\beta E_{empty}} + e^{-\beta E_{filled}}} \qquad P_{filled} = \frac{e^{-\beta E_{filled}}}{e^{-\beta E_{empty}} + e^{-\beta E_{filled}}}$$

$$\frac{P_{empty}}{P_{filled}} = \frac{e^{-\beta E_{empty}}}{e^{-\beta E_{filled}}} = \frac{e^{\beta(\frac{1}{2} g\mu_B B)}}{e^{-\beta U}} = e^{(U + \frac{1}{2} g\mu_B B)/kT}$$

$$= e^{(E_T - \mu + \frac{1}{2} g\mu_B B)/kT}$$