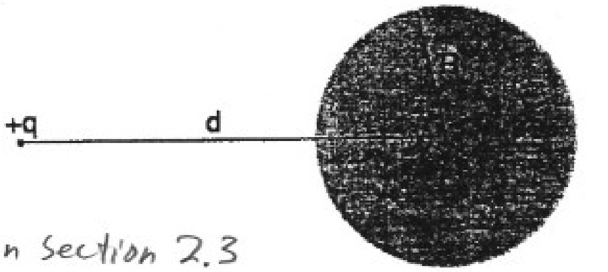
8. *Electricity and Magnetism* (Spring 2004)

A point charge q is located a distance d from the center of a conducting sphere of radius R. What must the total charge on the conducting sphere be for the force on the point charge to be zero?



See Jackson Section 2.3

First assume the sphere is grounded. Then we know by the method of images a charge $q' = -q\left(\frac{R}{d}\right)$ flows onto the sphere and distributes itself so that the field outside is like a point charge of charge q' at $x' = R\left(\frac{R}{d}\right)$.

Now remove the ground connection and add charge q'' to the sphere (which already has charge q' on it). Since the surface is an equipotential, the new charge will distribute itself uniformly over the surface, which is equivalent to an image charge of charge q'' at $x''=0$.

The field at $x=d$ is zero when

$$E(x=d) = \frac{1}{4\pi\epsilon_0}\left(\frac{q'}{(x'-d)^2} + \frac{q''}{(x''-d)^2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{-qR/d}{\left(\frac{R^2}{d}-d\right)^2} + \frac{q''}{d^2}\right) = 0$$

$$\Leftrightarrow \quad q'' = \frac{qRd}{\left(\frac{R^2}{d}-d\right)^2} = \frac{qd^3R}{(R^2-d^2)^2}$$

The total charge on the sphere is then

$$Q = q' + q'' = \frac{qd^3R}{(R^2-d^2)^2} - \frac{qR}{d} = \frac{qd^4R - qR(R^2-d^2)^2}{d(R^2-d^2)^2}$$

$$= qR\,\frac{2d^2R^2 - R^4}{d(R^2-d^2)^2}$$