4. *Quantum Mechanics* (Spring 2006)

Consider the scattering of a beam of non-relativistic spin 0 particles by a repulsive spherical potential of depth V_0 and radius a in three dimensions:

$$V(r) = \begin{cases} V_0, & \text{for } r < a; \\ 0, & \text{for } r > a. \end{cases}$$

Find the scattering cross section in the Born approximation.

The scattering cross section is $\sigma_{tot} = \int \frac{d\sigma}{d\Omega} d\Omega$

where $\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2$ and in the Born approximation

for spherically symmetric potentials, $f^{(1)}(\theta,\phi) = -\frac{2m}{q}\int_0^\infty r\sin(qr)V(r)dr$

where $q = 2K\sin(\theta_{sc}/2)$ and $\hbar K$ is the momentum transferred.

Our potential is $V(r) = V_0\Theta(a-r)$

$$\begin{aligned}
f^{(1)}(\theta,\phi) &= -\frac{2m}{q}\int_0^\infty r\sin(qr)V_0\Theta(a-r)\,dr \\
&= -\frac{2mV_0}{q}\int_0^a r\sin(qr)\,dr \\
&= -\frac{2mV_0}{q}\left[r\,\frac{-\cos(qr)}{q}\Big|_0^a - \int_0^a \frac{-\cos(qr)}{q}dr\right] \\
&= -\frac{2mV_0}{q}\left[-\frac{a\cos(qa)}{q} + \frac{\sin(qr)}{q^2}\Big|_0^a\right] \\
&= -\frac{2mV_0}{q}\left[-\frac{a\cos(qa)}{q} + \frac{\sin(qa)}{q^2}\right] \\
&= 2mV_0\left[\frac{a}{q^2}\cos(qa) - \frac{1}{q^3}\sin(qa)\right]
\end{aligned}$$

$$\frac{d\sigma}{d\Omega} = |f^{(1)}(\theta,\phi)|^2 = 4m^2V_0^2\left|\frac{a}{q^2}\cos(qa) - \frac{1}{q^3}\sin(qa)\right|^2$$

$$\sigma_{tot} = \int\frac{d\sigma}{d\Omega}d\Omega = 4m^2V_0^2\int\left|\frac{a}{q^2}\cos(qa) - \frac{1}{q^3}\sin(qa)\right|^2 d\Omega$$