Formula Functions

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March 19, 2004

Definition 0.1. A \textbf{formula function} is an entire function obtained by a finite composition of

(1) the parameter(s) of the function
(2) constant numbers
(3) functions from a finite set of elementary operators
(4) previously defined formula functions.

1 Conventions

The set of elementary operators will be \{+,-,\ast,\div,^,!\}. Factorial can only be used on integers. Exponentiation will be single-valued, so any square root (or even root) will return the positive root.

To make functions that return logical truth values, we need some way of generating discontinuities. It is now a common convention that $0^0 = 1$, which provides a discontinuity in the function $0^{|x|}$. This function is similar to the Kronecker Delta function with one parameter fixed as 0 and it is very useful for logic functions.

2 Formula Functions

\begin{align*}
abs(x) &= \sqrt{x^2} \\
\text{avg}(x, y) &= \frac{1}{2}(x + y) \\
\text{max}(x, y) &= \text{avg}(x, y) + \frac{1}{2}\abs(x - y) \\
\text{min}(x, y) &= \text{avg}(x, y) - \frac{1}{2}\abs(x - y) \\
\text{not}(x) &= 0^{\abs(x)} \\
\text{equ}(x, y) &= \text{not}(x - y) \\
\text{sig}(x) &= \frac{x}{\abs(x) + \text{not}(x)} \\
\text{pos}(x) &= \text{equ}(x, \abs(x)) \\
\text{neg}(x) &= \text{equ}(x, -\abs(x)) \\
gte(x, y) &= \text{equ}(y, \text{min}(x, y)) \\
lte(x, y) &= \text{equ}(y, \text{max}(x, y)) \\
\text{uni}(x) &= \text{gte}(x, 0) \ast \text{lte}(x, 1) \\
jag(x) &= x \ast \text{uni}(x)(1 - \text{equ}(x, 1)) \\
\ast\text{dec}(x) &= \Sigma_{n=-\infty}^{\infty} \text{jag}(x - n) \\
\text{int}(x) &= x - \text{dec}(x) \\
\text{aux}(x, y) &= \frac{y}{x + 2y \ast \text{not}(x) + \text{not}(x) \ast \text{not}(y)} \\
\text{div}(x, y) &= \text{equ}(\text{aux}(x, y), \text{int}(\text{aux}(x, y))) \\
\text{prm}(x) &= \text{div}(x, (x - 1)! + 1)
\end{align*}