Magnetism is not fundamental

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1 Introduction

It has been known for a while that the combination of Coulomb's Law and relativistic length contraction creates a force that fully accounts for what we call magnetism in situations where charges are moving parallel to a current. This result inspired some authors to call magnetism a relativistic effect, which implies that magnetism is not a part of the fundamental laws of the universe. However, other authors, noting the symmetry between the electric and magnetic fields in the original derivation, argued that electricity could also be called a relativistic effect. So the modern consensus seems to be that electricity and magnetism have equal footing in the fundamental laws of the universe. In this paper we will show why electricity is fundamental and magnetism is not.

There are actually two qualitatively different phenomena that we call magnetism. One occurs when charges move parallel to a current and the other occurs when charges move perpendicular to a current. The mechanism of the former phenomenon has been thoroughly explained with length contraction, but the mechanism of the latter phenomenon has never been explained before, so that is what we will try to do in this paper. The mechanism illustrated should make it clear that magnetism is not a fundamental law, but an emergent phenomenon.

To get a conceptual picture of the derivation, imagine a current that consists of a single charge flying down the x-axis from $x = -\infty$ to $x = \infty$ and a probe charge that moves down the y-axis toward the current. When the current charge is on the negative x-axis, it is farther from the probe charge than when it is on the positive x-axis. This is simply because the probe charge has moved closer to the x-axis while the current charge was travelling. So since the electric field strength drops with increasing distance, there will be a non-zero net horizontal impulse delivered to the probe charge. It is this impulse that accounts for magnetism because a current is just a continual repetition of this scenario.

Here is a brief outline of the derivation. We model a current by an infinite beam of evenly spaced electrons travelling with some constant velocity. But in order to simplify the math, we transform this current into beam of electrons travelling at the speed of light. We then calculate the net horizontal impulse delivered by the relativistic electric field of a single electron travelling at the speed of light along the entire x-axis. For this calculation, we assume that there is no such thing as magnetic fields or vector potentials by setting them all to zero. Then the resulting force is computed and compared to what Ampere's Law gives.

2 Impulse Calculation

In our model, a probe charge is moving perpendicular to an idealized infinite straight-line current. This current is modelled as an evenly spaced beam of electrons moving in the positive direction along the x-axis with constant velocity v_c. According to the forumla $I = \lambda v_c$, this current is equivalent to a current $I' = \lambda' c$ where $\lambda' = \frac{v_c}{c}\lambda$. We will make this replacement because it greatly simplifies the calculation, while admitting that this step requires rigorous justification, which can be found in the appendix. The reason why this current transformation is so helpful is that the electric field of a charge moving at the speed of light is compressed into a plane perpendicular to the direction of motion. Therefore, the field only interacts with the probe charge for an instant. However, in order to account for the motion of the probe, we will need to perform the calculation for some lower velocity and then take the limit as the velocity approaches the speed of light. If we just use the speed of light initially, the plane of the electric field will intersect the probe charge at one point and no motion of the probe charge will be registered.

We now calculate the impulse delivered to the moving probe charge by an electron travelling at the speed of light along the entire x-axis. Recall that will be assuming that the magnetic field and vector potential are zero.

$$V = \frac{q_c}{4\pi\epsilon_0}\frac{\gamma}{\sqrt{\gamma^2x^2+y^2+z^2}}$$

$$E_x = -\frac{\partial V}{\partial x} = \frac{q_c}{4\pi\epsilon_0}\frac{\gamma^3 x}{(\gamma^2x^2+y^2+z^2)^{3/2}}$$

Now we specialize to our dynamical model by letting $x = v_c t$ and $y = y_0 - v_p t$ and $z = 0$.

$$E_x(t) = \frac{q_c}{4\pi\epsilon_0}\frac{\gamma^3 v_c t}{(\gamma^2 v_c^2 t^2 + (y_0 - v_p t)^2)^{3/2}}$$

$$\begin{aligned} J = \lim_{v_c\to\infty}\int_{-\infty}^{\infty} q_p E_x(t)\ dt &= \frac{q_c q_p}{4\pi\epsilon_0}\lim_{v_c\to\infty}\int_{-\infty}^{\infty}\frac{\gamma^3 v_c t}{(\gamma^2 v_c^2 t^2 + (y_0 - v_p t)^2)^{3/2}}\ dt \\ &= \frac{q_c q_p}{4\pi\epsilon_0}\lim_{v_c\to\infty}\frac{2\gamma v_p}{v_c y_0\sqrt{\gamma^2 v_c^2 + v_p^2}} \\ &= \frac{q_c q_p}{4\pi\epsilon_0}\frac{2 v_p}{c^2 y_0} \end{aligned}$$

3 Comparison to Maxwell's Equations

Now we can compute the horizontal component of the electric force on the probe charge through the impulse delivered by the current. Let J_1 be the total impulse delivered to the probe charge during a single iteration, where an iteration occurs after each charge in the beam traverse the inter-charge gap distance. Based on the argument in the appendix, we know that the impulse delivered during each iteration is the same as the impulse delivered by a single electron travelling at the speed of light, as computed in the last section. The number of iterations per second is given by the number of charges per meter times the length travelled per second. So if we let λ be the charge per unit length, then the number of iterations per second is $f = \lambda v_c/q_c$. Therefore, the total impulse delivered to the probe charge during a time interval of length t is

$$J(t) = tfJ_1 = t\lambda v_c J_1/q_c$$

$$F = \frac{dJ}{dt} = \frac{\lambda v_c}{q_c}\left(\frac{q_c q_p}{4\pi\epsilon_0}\frac{2v_p}{c^2 y_0}\right)$$

$$F = \frac{\lambda q_p}{4\pi\epsilon_0}\frac{2 v_p v_c}{c^2 y_0}$$

Now we would like to compare this result to what the standard Maxwell's equations would give us. By Ampere's Law, the magnetic field at a distance y_0 from a current-carrying wire has magnitude

$$B = \frac{\mu_0 I}{2\pi y_0}$$

where r is the distance I is the current in the wire. The current can be expressed in terms of the linear charge density and velocity by

$$I = \lambda v_c$$

and we can eliminate μ_0 using the relation

$$\epsilon_0\mu_0 = \frac{1}{c^2}$$

$$B = \frac{\lambda v_c}{2\pi\epsilon_0 c^2 y_0}$$

According to the Lorentz force law, we have

$$F = q_p v_p B = \frac{\lambda q_p}{4\pi\epsilon_0}\frac{2 v_p v_c}{c^2 y_0}$$

This is the exact same expression that was obtained by assuming that there is no magnetic field and computing the horizontal component of the relativistic electric field.

4 Conclusion

If magnetism can be fully explained as a relativistic effect, then there is no reason for us to believe that magnetism is at all fundamental. Similarly, there is no more likely that we will find magnetic monopoles than any other imagined particle. Nonetheless, magnetism is an incredibly useful mathematical tool. It allows one to compute the electric force on a probe without accounting for its motion through space.

5 Appendix: Equivalence of speed of light current

As is well known, the electric force propagates at the speed of light. So when a charge moves, there is no way for any other charge to be aware of this motion until after a duration corresponding to the time it takes for light to travel the distance between the two charges. The region of space that is aware of the new position is a sphere, centered around the moved charge, whose radius is expanding at the speed of light. When this sphere intersects another charged particle, there will be an impulse delivered to the particle due to the new relative configuration of the two charges.

In fact, this process of expanding "impulse spheres" can also explain forces between stationary charges if we assume that impulse spheres are generated at regular intervals. Then the strength of the force corresponds to the number of impulse spheres intercepted per second, weighted by the magnitude of the delivered impulse for each sphere. Of course the magnitude of the delivered impulse must drop with the square of the radius of the sphere because it is being spread over a larger surface area. For simplicity, we will assume that each impulse sphere starts with the same amount of deliverable impulse, so that the radius is the only variable that affects the magnitude of the delivered impulse. This just means that rather than having an impulse sphere that is twice as strong, we imagine that two impulse spheres were created instead.

For the purposes of discussing magnetism, we are solely concerned with impulse spheres created during the motion of charge. The main question is whether the velocity of a charge affects the rate of emission of impulse spheres (or equivalently, whether it affects the strength). We can do a thought experiment where we can control the positions of particles arbitrarily. Assume that we have two identical massless charges so that we can accelerate them instantaneously. The first charge we move from point A to point B continuously during a period T. The second charge we move between point A and point B during the same period T, but in a jagged fashion. We move the second charge at twice the velocity for a short time dt, then hold it still for the same time dt. From the viewpoint of a distant observer, both of these situations are identical, so they should deliver the same impulse. Since we are considering only impulse sphere due to motion, we know that the second particle spends half its time not emitting any relevant impulse spheres. Therefore it must be emitting impulse spheres at twice the rate of the first charge during the intervals when it is moving. So we can conclude that impulse spheres due to motion are released at a rate proportional to the distance travelled, not the time elapsed.

In our model, a probe charge is moving perpendicular to an idealized infinite straight-line current. This current is modelled as a cathode ray beam of electrons moving in the positive direction along the x-axis with velocity v_c. Let δx_c be the distance between adjacent electrons. Our goal is to calculate the horizontal component of the electric force on the probe charge due to this current. We will show that the result is what is known as the "magnetic force".

The math is rather difficult if we use the most straightforward method. It will be much easier if we perform a transformation on the current. We will show that a beam of electrons travelling at the speed of light can have the exact same impulse sphere distribution if the spacing is chosen appropriately.

Now we already showed that impulse spheres due to motion are emitted at regular intervals in space. We can choose this interval arbitrarily because scaling the interval size just scales the impulse sphere's magnitude correspondingly. The trick is to choose the interval length to be the distance each electron travels in the time it takes light to travel the length δx_c. So impulse spheres are emitted whenever an electron travels a distance $\delta x_e = \frac{v_c}{c}\delta x_c$. This means that each impulse sphere reaches the next electron in the beam just as that electron is ready to emit its next impulse sphere. Therefore, whenever an electron emits an impulse sphere, it is also being intersected by the last impulse sphere from the previous electron, and the second to last impulse sphere from the electron before the previous electron, etc. Actually, at locations where impulse spheres are emitted, there are also impulse spheres reaching that point from all electrons that are further back in the beam.

Consider the impulse spheres that pass through a given point on the x-axis during the course of one iteration (one iteration occurs when each electron travels the gap distance). After a time $\delta x_e/c$, there will be one impulse sphere for each electron to the left on the x-axis. These each correspond to the motion of an electron across a distance δx_e. After one full iteration, there will be an impulse sphere for each electron and for each δx_e interval in the gap length. In the limit of small gap size, i.e. $\delta x_c \rightarrow 0$, we have the impulse spheres for each motion step along the entire x-axis crossing at the same time. This is exactly what happens for a particle travelling at the speed of light because its impulse spheres bunch like a shock wave. Therefore, the current executing one iteration is equivalent to a single electron travelling at the speed of light along the entire x-axis. Note that if it is equivalent at one point on the x-axis, then it must be equivalent at all points in space because we have the same center and radius, which fully specifies a sphere.

There is one subtle point here. The reader may have noticed that the probe charge actually traverses the entire y-axis according to this model (since it has constant velocity v_p and the process takes infinite time), which is not what was happening in the current beam model. So one might think that our calculation will be off due to the differing distances between the two charges. However, relativistic gamma factors cause a contraction of the electric field in the x-direction. So in the limit $\gamma \rightarrow \infty$, all the impulse hits the probe charge at the same time, which is exactly when the current charge reaches $x = 0$ and the probe charge reaches $y = y_0$. Naturally, we have to take the limit rather than evaluating at $\gamma = \infty$ or the effect of the motion of the probe charge will be lost.