The quantum topological phase shift known as Berry’s phase is measured using a laser coupled into a helically wound optical fiber. In the context of this experiment, Berry’s phase corresponds to rotations in the plane of polarization of the laser beam due to an adiabatic transport of the beam’s momentum vector through a closed loop. Although parts of the experiment were awash in errors, valid data was obtained by utilizing ad hoc methods which permitted the substantiation of Berry’s claim.

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INTRODUCTION

Berry’s Phase is an effect caused by the continuous transport of a quantum system around a closed loop in parameter space. Although the quantum state of the system is guaranteed to return to its initial state in any adiabatic process, the phase can actually be modified. In our experiment, the phase corresponds to the polarization of a laser beam.

The polarization of light is defined to be the path traced by the electric field vector in the plane perpendicular to the direction of propagation. A polarizer is a device that emits only light with a fixed linear polarization axis. Light that does not have this exact polarization axis can still be transmitted, but with a probability that decreases with increasing angular difference. When a polarizer is placed in a linearly polarized beam of intensity $I_0$ with its axis at an angle $\theta$ with respect to the beam’s polarization axis, the transmitted intensity is given by Malus’s Law:

$$I = I_0 \cos^2(\theta)$$

This phase shift can be understood in terms of the concept of parallel transport.

Parallel transport refers to the phenomenon by which a vector tangent to a surface can be transported along a path such that it never rotates relative to the path, but undergoes a net rotation by the time it returns to its starting location. As a simple illustration, consider a north-pointing vector on the equator of a sphere. Let this vector slide along a line of longitude to the north pole. Without rotating the vector, let it slide back down to the equator along a line of longitude that is 90° from the first line. Finally, let the vector slide along the equator to its initial location. It will now be pointing along the equator instead of towards the north pole. In laser experiments, after some region of topological modification, the beam path assumes the same tangent direction as the incident beam, but with a new shift in the plane of polarization that we refer to as Berry’s Phase.

FIG. 1: Solid angle in momentum space

Berry’s central claim is that the angle of rotation of the plane of polarization per cycle is equal to the solid angle circumscribed by the beam tangent vector in momentum space. The shaded region shown in Figure 1 depicts the solid angle circumscribed by the tangent to the helix laying on the inscribed cylinder. To control the tangent of the beam, it is passed through a fiberoptic cable. For the case of a uniform helix of pitch angle $\theta$, the solid angle can be calculated from the definition of solid angle over a region of a sphere

$$\Omega = \int_S \sin(\theta) d\theta d\phi = 2\pi \int_0^\theta \sin(\theta') d\theta' = 2\pi(1 - \cos(\theta))$$

Therefore, for a helix of $N$ turns, the phase shift in radians will be

$$\Delta\phi = 2\pi N (1 - \cos(\theta))$$

This phase shift can be understood in terms of the concept of parallel transport.

EXPERIMENTAL APPARATUS AND MEASUREMENT

The experimental apparatus consists of a He-Ne laser, a single mode non-polarization preserving fiber, and various optical accessories as shown in Figure 2. An iris diaphragm was used to reduce polarization fluctuations in
the beam, either by allowing a more efficient coupling into the mode of the fiber or by blocking out back-reflected light [5]. Two fiber supports were also used to keep the fiber at a constant height above the table throughout the fiber’s path. A cylindrical tube is used to hold the fiber into a helical shape.

FIG. 2: Schematic of the experimental apparatus

The beam passes through a polarizer and is coupled into an optical fiber. The fiber is wound into a helical shape and continues to the output fiber coupler. After exiting the fiber, the beam passes through a rotating polarizer, hereafter referred to as a polarization analyzer. After passing through this polarization analyzer the beam is imaged onto a photodiode whereby the intensity is measured using an oscilloscope, assuming linearity of the photodetector’s voltage with incident intensity. We do not need to assume that the photodetector is linear, but we do assume that its output is nondecreasing with increasing incident intensity and vice versa.

In order to take relevant and useful data, the beam must be coupled to the fiber as efficiently as possible. In order to do this, mirrors are used to “walk” the beam into the coupler. Due to the extremely small aperture of the fiber, fine tuning of the mirrors is necessary to achieve maximum coupling efficiency. By measuring the intensity of the beam before and after its traversal through the optical fiber, we were able to obtain a maximum efficiency of about 10%.

Measurements of the intensity of the beam were taken for various helical pitch angles of the fiber. As a result of Malus’ Law, the output voltage oscillates with the difference between the polarization of the beam and the rotation angle of the polarization analyzer. By rotating the analyzer we were able to obtain a \( \cos^2 \) curve of voltage versus rotation angle of the polarization analyzer for each pitch angle. Berry’s phase is responsible for phase shifts in these curves with respect to the curve corresponding to a straight fiber.

Several issues became apparent during the experiment. The first of which was an erratic fluctuation in the beam intensity that appeared after a polarizer. We attributed this to random oscillations in the linear polarization of the laser that occurred with a period of approximately 15 seconds [6]. A second type of fluctuation was characterized by more volatile perturbation and was attributed to noise created by heating of the laser and acoustic disturbances. Both forms of fluctuation were deemed consequential to our data because our major concern was determining the location of the extrema with respect to the rotation angle of the polarization analyzer. Since the height of the \( \cos^2 \) wave is irrelevant to this calculation, we were able to neglect the effects and data was taken with impunity.

A second but more important issue occurred when the experiment was run on different days using the same configuration. Measurements taken on different days resulted in completely different locations of transmission minima on the analyzer’s scale. This is a debilitating problem because it implies the non-repeatability of the experiment. To expedite the data-taking process and obtain more data, we chose to find only the angles of minimal transmission through the analyzer for each pitch angle.

The primary variable between the different setups is the specific spatial arrangement of the fiber. The arrangement causes not only Berry’s phase, but also additional shifts due to torsional stress and fine details of the arrangement. Torsional stress applied to the fiber induces a polarization shift due to the elasto-optic effect [1]. Unfortunately, it seemed that the results were dominated by these other factors. Therefore, the fine details of the fiber arrangement constitute a non-measurable variable that has a measurable and substantial effect on the data.

RESULTS AND ANALYSIS

A typical set of measurements is shown in Figure 3. Each data point is subject to a \( \pm 5\% \) error. The expected curve is obtained by applying the expected Berry’s Phase shift to the curve generated using a straight fiber. The intensities found at 5 degree intervals clearly establishes the periodicity of the polarizing filter. However, it is immedi-
ately evident that the experimental result is far from the expected value. In fact, the error in the measurements is sufficiently dreadful that it is unclear whether the results contain any meaning. Table 1 provides the polarization shifts (in degrees) with respect to a straight-line fiber path for each configuration of the form $N \times \theta$, where $N$ is the number of revolutions and $\theta$ is the pitch angle of the helix expressed in degrees. Note that this is for one particular run, and other runs had very different values. Each data point is subject to a $\pm 5$ degree error. Expected values were computed using equation 2 and the best case scenario is assumed i.e. expected values are reduced modulo 180$^\circ$ to the point that is closest to the measured quantity. Note that it is possible that the results are off by 180 degrees more, but it is unreasonable to make that assumption because there is no reason to assume that the data is worse than it has to be. The difference between the measured and expected values is given, where the worst possible difference is 90$^\circ$ due to the best case scenario assumption.

<table>
<thead>
<tr>
<th>Setup</th>
<th>Measured</th>
<th>Expected</th>
<th>Difference</th>
</tr>
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<td>1x50</td>
<td>-91</td>
<td>-51</td>
<td>40</td>
</tr>
<tr>
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<td>-42</td>
<td>0</td>
<td>42</td>
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</tr>
<tr>
<td>2x85</td>
<td>-50</td>
<td>-63</td>
<td>13</td>
</tr>
</tbody>
</table>

**Table 1 Polarization shifts**

Due to the lackluster performance of the original set of configurations, we chose to perform additional experiments with $N > 2$ revolutions of the fiber. In some cases, this provided much better results. All configurations that fit onto the cylinder were tried, which included all those listed in Table 2. With the blatant exception of 4x80 and 4x85, the difference between the measured and expected shifts was less than 10 degrees.

<table>
<thead>
<tr>
<th>Setup</th>
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<th>Difference</th>
</tr>
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<tbody>
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<td>19</td>
<td>23</td>
<td>4</td>
</tr>
</tbody>
</table>

**Table 2 Polarization shifts**

The fact that more revolutions gives better results provides some indication of the primary source of error in the experiment. At first it seems natural that more repetitions should yield more accurate measurements. This is because it is easy to assume that the configuration is performing an automatic averaging of the contributions from each revolution, and hence diminishing the errors. However, no averaging occurs; the contributions are merely added and then modularly reduced by $\pi$, with the critical step of dividing by $N$ missing. The situation can be modelled by considering each of the $N$ revolutions as an independent trial with the final measured result being the sum of the random variables from each of the $N$ trials.

Now, if the error was systematic, such as a constant shift due to torsion in the wire, then it will propagate through any number of revolutions without being affected since each revolution just adds on its own phase shift. So the addition of more revolutions does not directly reduce systematic error.

On the other hand, if the error was random, such as in the case that each revolution contributes a random perturbation, then the final result will have its own error distribution due to the “sum” of the individual errors. The term “sum” actually refers to the distribution that has for each point the total probability of obtaining the error corresponding to that point. This total probability is calculated by summing all the probabilities of combinations of error from the individual trials that sum to the required total. This operation is called convolution, and since each trial is the same, it is actually autoconvolution. Therefore, for a sequence of $N$ trials, the random error in the sum is the $N^{th}$ order autoconvolution of the error from an individual trial. We will now assume that the error is Gaussian, though the final result will hold for more general error distributions. The convolution of two Gaussian curves $f$ and $g$ with standard deviations $\sigma_f$ and $\sigma_g$ is another Gaussian curve with standard deviation $\sqrt{\sigma_f^2 + \sigma_g^2}$ [4]. Therefore the autoconvolution of $f$ has standard deviation $\sqrt{N}\sigma_f$. More generally, the $N^{th}$ order autoconvolution of $f$ has standard deviation $\sqrt{N}\sigma_f$. Thus the random error found in the sum of the trials is spread more than that in an individual trial. The process of modularly reducing the result by $\pi$ just shifts all quantities, which does not decrease this spread in error. Also, the relative size of the real contribution from Berry’s Phase is inconsequential since we only measure the total shift. Therefore, the random error tends to increase with the square root of the number of trials [7].

These observations rule out the common random and systematic error, but notice that these observations do not rule out the possibility of systematic error that is indirectly caused by introducing more trials. This could be the case, say, if more revolutions somehow minimize the torsional stress in the fiber. Another example is if lengths of straight fiber create more systematic error than curved lengths. Any special phenomenon that can somehow dis-
cern the number of revolutions could also be a possible cause.

CONCLUSION

With the outcome of our experiment, we are incapable of asserting the validity of Berry’s phase relation. However, part of our experiment was consistent with Berry’s phase to the extent that it is improbable that the correlation was coincidental. Therefore, we believe that there is some truth behind Berry’s claim.

Additionally, we discovered an interesting connection between the accuracy of the data and the number of revolutions of the fiber. Since the relationship has been shown to be non-statistical, it is plausible that it has physical or geometrical significance.

ACKNOWLEDGEMENTS

We would like to thank Professor John Howell and Michaela Tscherneck for their magnanimous advice and support.

[5] Coupling into the fiber can be improved by an iris diaphragm since the tail of the gaussian width of the beam is chopped off.
[6] It may be possible to minimize these oscillations by inserting a spatial depolarizer before the first polarizer. In this setup, the polarization of the beam will be randomized before passing through the polarizer, so that the transmitted intensity will be independent of the initial polarization. As a side effect of the spatial depolarizer, the beam after the polarizer will be split into a few thousand fringes that alternate in intensity based on the laser’s polarization. But the integrated intensity over any region larger than a few square micrometers should be constant.
[7] In the case of averaging, the error is divided by \(N\), so the random error has standard deviation \(\sigma_f/\sqrt{N}\).