DRAFT - Magnetism is Not Fundamental

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INTRODUCTION

The unification of electricity and magnetism is well known, but the extent to which they are unified is not. Magnetism is just a natural consequence of Coulomb's Law and the finite speed of propagation of the electric force (the speed of light). This already sounds a lot like special relativity, but special relativity is not the whole story. Special relativity is too abstract to give the result that magnetism is caused by electricity rather than the other way around. This paper provides a low-level explanation of the mechanism by which the electric force produces the phenomenon that we know as magnetism.

THE ELECTRIC FIELD

The first step is to compute the actual electric field of a moving charge. For the purposes of illustration, pretend that time is quantized and the universe jumps from one state to the next on every time-step. To picture the electric field, imagine that every charged particle emits a spherical signal at every time-step (the outermost of these is commonly referred to as a light-cone). Whenever one of these spheres intersects another charged particle, a force is exerted on the intercepted particle. The electric field calculation will proceed under the assumptions that the magnitude of the delivered force is given by Coulomb's Law and that the radii of the spherical signals increase at the speed of light. Both of these assumptions are well established, but there are some interesting consequences. The effect of the finite propagation speed is to cause the field to look like the charge is at an old location. This is because it takes some time for the signal to reach the location of the observer, and during this time the charge moves. The concept of retarded time deals with this same idea, but it is not usually applied to the electric field because doing so violates Maxwell's Equations. Therefore, it will be necessary to forget about Maxwell's Equations for the time being.

To begin calculating the electric field, it is necessary to find expressions for the effect of retardation on the field. Suppose an electron is traveling along the x axis in the $+x$ direction with velocity v. Figure 1 represents this situation. This applies to three dimensions by utilizing the cylindrical symmetry (x is axis and y is radial). Every point in space is getting a signal that departed from the electron at some time in the past. Considering a specific point in space, the length of this time interval will be denoted δt. Say that the position of the electron at $t-\delta t$ was $x-\delta x$. Since the signal always travels with velocity c, and the distance that the signal traveled was r', it follows that

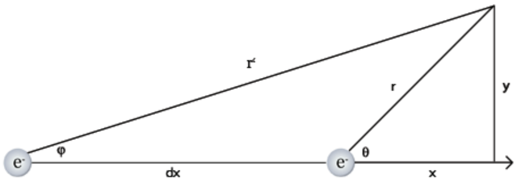

FIG. 1: Retarded position

$$r' = c\delta t. \tag{1}$$

Given that the velocity of the electron is constant and equal to v, it is also true that

$$\delta x = v\delta t. \tag{2}$$

Finally, by the Pythagorean theorem,

$$(r')^2 = (x+\delta x)^2 + y^2. \tag{3}$$

These constitute all of the observations that are needed to determine the effects of the retarded field.

Plugging (1) and (2) into (3) yields

$$c^2\delta t^2 = (x+v\delta t)^2 + y^2$$

$$(c^2-v^2)\delta t^2 - (2vx)\delta t - (x^2+y^2) = 0$$

Now using the quadratic equation,

$$\delta t = \frac{1}{2(c^2-v^2)}\left[2vx \pm \sqrt{4v^2x^2 + 4(c^2-v^2)(x^2+y^2)}\right]$$

Rearranging,

$$= \frac{1}{c^2-v^2}\left[vx \pm \sqrt{\cancel{v^2x^2} + c^2x^2 - \cancel{v^2x^2} + c^2y^2 - v^2y^2}\right]$$

$$= \frac{vx + \sqrt{c^2x^2 + (c^2 - v^2)y^2}}{c^2 - v^2}$$

The plus sign was chosen because otherwise δt would be negative, which would make r' negative, but it is a length.

The components of the retarded field can now be computed by invoking Coulomb's Law. The magnitude is given by

$$R = \frac{1}{4\pi\epsilon_0}\frac{q_e}{(r')^2} = \frac{1}{4\pi\epsilon_0}\frac{q_e}{(c\delta t)^2}$$

The components are

$$R_x = Rcos(\varphi) = R\frac{x + v\delta t}{c\delta t}$$

$$R_y = Rsin(\varphi) = R\frac{y}{c\delta t}$$

These are not the correct equations for the electric field—there is an additional subtlety here. The electric field actually undergoes a form of Doppler shifting that scales the magnitude of the force. This effect results from the fact that the spherical signals grow at the same rate in all directions, but the location where new spheres are being generated is moving with the particle. Figure 2 depicts this situation with some representative circles.

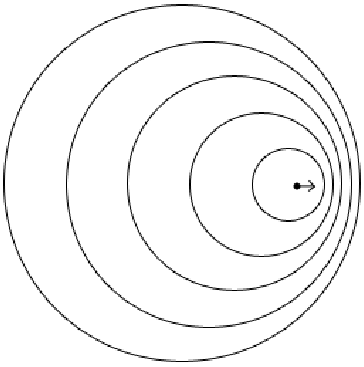

FIG. 2: Doppler shifting of the electric field

The compression of spheres results in a scaling of the electric force's strength because each spherical signal delivers a certain quantity of force. A higher density of spherical signals will deliver a larger amount of force in the same amount of time. It is important to notice that consecutive signals will have approximately the same direction and magnitude, so the strength is nearly linear with density. This observations permits a trick for calculating the scale factor. The idea is to consider a short interval of time and calculate how many signals were created in the past that should be intercepted during that interval. Since the particle always travels at the constant velocity v, the number will be proportional to the length of its path that the retarded particle travels in that same interval. 'Retarded particle' is a term used to refer to an imaginary particle that is located at $x - \delta x$, which is where the particle looks like it is. The length traversed by the retarded particle during the interval is in turn proportional to the velocity of the retarded particle, which is not constant—it depends on how close it is to the observer. Some simple algebra will provide the proper constant of proportionality. Let r be the number of time-steps per second. Then the original count of signals received is $c_0 = rt$. As shown, the corrected count is

$$c = \frac{\Delta x'}{v}r,$$

where $\Delta x'$ is the length of path that the retarded particle travels in the time interval. Now, $\Delta x' = v't$ where v' is the velocity of the retarded particle at the point under consideration, so

$$c = \frac{v'}{v}rt = \frac{v'}{v}c_0.$$

Therefore, the required scale factor is

$$D = \frac{v'}{v}. \tag{4}$$

To evaluate this expression for D, it is easiest to use the frame of reference of the observer and define $x_p = -x$ to be the location of the moving particle. It is equal to the negative of the previous coordinate x since that was the position of the observer relative to the moving particle. Let $p(x_p)$ be the location of the retarded particle when the particle is at x_p. Then

$$x_p - p(x_p) = v\delta t.$$

The scale factor can now be evaluated:

$$D = \frac{v'}{v} = \frac{1}{v}\frac{dp(x_p)}{dt} = \frac{1}{v}\frac{dp(x_p)}{dx_p}\frac{dx_p}{dt} = \frac{dp(x_p)}{dx_p}$$

$$= \frac{d}{dx_p}(x_p - v\delta t) = 1 - v\frac{d(\delta t)}{dx_p} = 1 + v\frac{d(\delta t)}{dx}$$

$$= 1 + v\frac{d}{dx}\left(\frac{vx + \sqrt{c^2x^2 + (c^2 - v^2)y^2}}{c^2 - v^2}\right)$$

$$= 1 + v\frac{v + \frac{c^2x}{\sqrt{c^2x^2+(c^2-v^2)y^2}}}{c^2 - v^2}$$

$$= \frac{c^2 - v^2}{c^2 - v^2} + \frac{v^2 + \frac{c^2xv}{\sqrt{c^2x^2+(c^2-v^2)y^2}}}{c^2 - v^2}$$

$$= \frac{c^2 + c^2 \frac{xv}{\sqrt{c^2x^2+(c^2-v^2)y^2}}}{c^2 - v^2}$$

The field of a moving charge is thus

$$E_x = DR_x$$

$$E_y = DR_y$$

Figure 3 shows a vector plot of the field of a positive charge traveling rightwards at 8/10 the speed of light (arrows are not to scale).

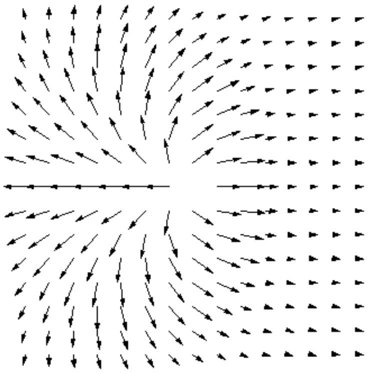

FIG. 3: Electric field of a charge moving to the right at 0.8c

MAGNETISM

At last it is time to see how this field creates magnetism. The connection is not simple—the most obvious way of demonstrating the effects of this field is to integrate the force over some path to find the net force. This is not a line integral because there is no projection along the direction of integration, so this is not a work integral. The radial (y) component of the force is unnecessary; it will be zero in any normal current because the stationary positive charges will cancel it. Therefore, only the axial (x) component of the force will be considered.

The first thing to notice is that if the moving particle goes from $-\infty$ to ∞, then any particle sitting near the origin will feel zero net force in the x direction. This can be seen from the fact that spherical signals are emitted at regular spatial intervals, and eventually all will be intercepted. Since there are an equal number coming from both sides, the final net force in the x direction will be zero. This is consistent with the fact that a stationary charge feels no magnetic force.

An understanding of the features of the field is necessary to understand what happens. As seen in figure 3, there is a column of vectors that are pointing strongly to the right. This column delivers the majority of the force that came from the moving particle while it was to the left of the observer. Effectively, the force piles up and gets dumped onto the observer during its presence in this short column. So if an observer is not around when this column goes by, but shows up later, then they will miss most of the effect of the particle from that half of space. In reality, observers cannot just appear out of nowhere, but traveling toward the axis of propagation while the particle is going by has the same effect. Therefore, a charge with radial velocity about the x axis will feel a nonzero net charge in the x direction. The rightward column always comes first, so a particle moving toward the axis will miss some force in the $+x$ direction and thus feel a net force in the $-x$ direction and vice verse. This is qualitatively consistent with the right hand rule.

So in a normal case, such as a current in a wire, there is a net flow of electrons in one direction. When a charged particle moved radially with respect to the wire, these electrons each contribute a tiny net force in the x direction, and because there are so many of them, they create a noticeable force. A current also has the property that the force instantaneously cancels for a stationary particle because there are an equal number of electrons on either side that cancel each other. So, there is no need to go all the way from $-\infty$ to ∞ for the net force to go to zero. It also turns out that the net force is linear with velocity for low velocities, which is consistent with the fact that the strength of the classical magnetic field is proportional to the current.

COMPUTATION

It is helpful to perform some numerical estimates to check that the equations work out. For convenience, take the unit of distance to be 1 light-second so that $c = 1$. The strategy is to approximate the moving charge as a steady current and determine what net force is predicted by the Lorentz force law and then compare to an integral of the electric field. A path that represents a trajectory with a purely radial velocity becomes a diagonal line on the field plot. The example will have a positive test charge at $(0, 2)$ until the moving positive charge is at $(-2, 0)$ where it will proceed to $(0, 1)$ by the time the moving charge is at $(2, 0)$. Therefore, the test charge will have radial velocity $v_r = v/4$. The magnetic field for a line current is

$$B = \frac{\mu_0 I}{2\pi r}$$

and it is perpendicular to the radial direction, so the Lorentz force law says

$$F = q(v_r \times B) = qv_r \frac{\mu_0 I}{2\pi r}$$

Making the approximation that the moving charge represents a current with one unit of length between each charge, $I = qv$. Integrating F over r,

$$NetForce = q^2 \frac{\mu_0}{2\pi} v v_r ln(2) = 5.076 * 10^{-29}$$

Performing an integral along this path on E_x from $x = -1000$ to $x = 1000$ yields $4.667 * 10^{-29}$, which is a reasonable amount of error given the nature of the approximation made.

CONCLUSION

The conclusion is: Magnetism is not fundamental. What does it mean for a phenomenon to be fundamental? Suppose you had to write a universe simulator computer program. If you must explicitly program a specific phenomenon, then that phenomenon is fundamental, otherwise it is not fundamental. For example, friction is not fundamental because the rule for the electric force covers it. The same applies to magnetism, so magnetism is not fundamental either. This is important because it simplifies the theoretical basis of physics.