4. *Quantum Mechanics* (Fall 2002)
Consider the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{r}$$

(a) What is the ground state energy of this Hamiltonian?

(b) What is the expectation value of the potential energy $\left\langle -\frac{Ze^2}{r} \right\rangle$ in the ground state?

(c) What is the expectation value of the kinetic energy $\left\langle -\frac{\hbar^2}{2m}\nabla^2 \right\rangle$ in the ground state?

a. This is the Hydrogen-like atom Hamiltonian
so $E_n = -\frac{mZ^2e^4}{2\hbar^2 n^2} \Rightarrow$ Ground state is $E_1 = -\frac{mZ^2e^4}{2\hbar^2}$
(or $E_n = -\frac{Z^2e^2}{a_0 n^2}$ or $E_n = -\frac{Z^2\alpha^2}{2n^2}mc^2$)

b. Use the virial theorem for central potentials.
$\langle V \rangle = -2\langle T \rangle \Rightarrow E_1 = \langle T \rangle + \langle V \rangle = -\langle T \rangle = \frac{1}{2}\langle V \rangle$
$\Rightarrow \langle V \rangle = 2E_1 = -\frac{mZ^2e^4}{\hbar^2}$

c. $\langle T \rangle = -E_1 = \frac{mZ^2e^4}{2\hbar^2}$